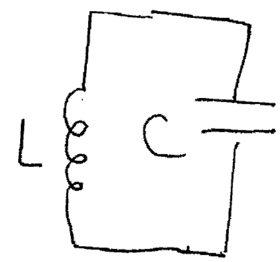
LC - time domain analysis :

We know LC circuits oscillate, lets prove it using conservation of Energy.

$$V_L + V_c = 0$$



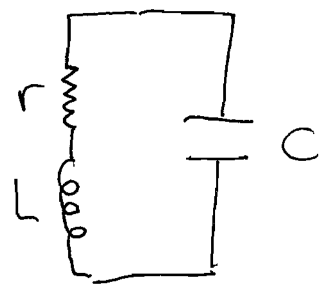
$$L \underbrace{\frac{dI}{dt}}_{\frac{d^2Q}{dt^2}} + \frac{Q}{C} = 0$$

$$\frac{d^2Q}{dt^2} = -\frac{1}{LC} Q \quad , \quad \boxed{Q = Q_0 e^{i\omega t} \quad \omega = \frac{1}{\sqrt{LC}}}$$

$$\boxed{V_c = \frac{Q_0}{C} e^{i\omega t}}$$

$$\boxed{V_L = -\omega^2 L e^{i\omega t}}$$

The previous equations are true, but we always see loss in our systems due to resistance.



$$V_r + V_L + V_c = 0$$

$$Ir + L\frac{dI}{dt} + \frac{Q}{C} = 0$$

$$r\frac{dQ}{dt} + L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0$$

rearange...

$$\ddot{Q} + \frac{r}{L}\dot{Q} + \frac{Q}{LC} = 0 \qquad \left(\dot{Q} = \frac{dQ}{dt}\right)$$

Solve using a root method

Assume $Q(t) = e^{\alpha t}$, plug this in along w/ the correct derivatives and solve for α.

$$\ddot{Q} + \frac{r}{L}\dot{Q} + \frac{Q}{LC} = 0$$

Ansatz: $Q(t) = e^{\alpha t}$

$$\alpha^2 e^{\alpha t} + \frac{r}{L}\alpha e^{\alpha t} + \frac{e^{\alpha t}}{LC} = 0$$

or

$$\alpha^2 + \frac{r}{L}\alpha + \frac{1}{LC} = 0$$

Solve for α w/ quadratic formula.

$$\boxed{\alpha = \frac{-\frac{r}{L} \pm \sqrt{\left(\frac{r}{L}\right)^2 - \frac{4}{LC}}}{2}}$$

$$\boxed{Q(t) = e^{\alpha t}}$$

Check:

when $r \to 0$ $\alpha = \frac{i}{\sqrt{LC}}$, which matches our first result.

when $\underline{r \to \infty}$ decay becomes dominant and $Q = e^{-\frac{r}{L}t}$.

The point at which oscillation is no longer seen is found by looking at when α ceases to be complex, when

$$\frac{r}{L} > \frac{2}{\sqrt{LC}}$$ no more

ringing occurs.